**Polynomials**

**Fundamental Theorem of Algebra (v1)** Every non-constant polynomial has a root (in ).

**Fundamental Theorem of Algebra (v2)** Every non-constant polynomial can be factored uniquely into a product of irreducible polynomials.

Notice that this theorem is strikingly similar to another important theorem we’ve discussed. What are the similarities (and the differences) with the Fundamental Theorem of Arithmetic?

Use Maple to factor the following polynomials:

1. x2 + 2x – 15

1. x2 + x + 1
2. x3 – 2x2 – 2x – 3

1. x5 – 5x4 – 28x3 + 100x2 – 29x + 105

What do you think the “irreducible polynomials” are that the Fundamental Theorem of Algebra talks about?

Try factoring these polynomials again, but adding the option “complex” to the command (that is try **factor(x2+2x-15, complex);** etc).

1. x2 + 2x – 15

1. x2 + x + 1

1. x3 – 2x2 – 2x – 3

1. x5 – 5x4 – 28x3 + 100x2 – 29x + 105

What difference did adding “complex” make?

 If we allow complex numbers, what kinds of “irreducible polynomials” do we get?

How does the number of factors relate to the degree of the polynomial?

If we can factor polynomials, then it makes sense to talk about their gcd’s and lcm’s. Try finding the gcd and lcm of the following:

1. x2 + 2x – 15 and x3 – 2x2 – 2x – 3
2. x5 – 5x4 – 28x3 + 100x2 – 29x + 105 and x3 + 5x2 + x + 5
3. x5 – 5x4 – 28x3 + 100x2 – 29x + 105 and x2 + x + 1

Do you think gcd(m,n) \* lcm(m,n) = m\*n is still true even if m and n are polynomials instead of numbers? Prove or disprove it.